Chelsea Smith
MTH 201-05
Portfolio Problem 1

Portfolio Problem

Problem 1: Section 1.3 #2

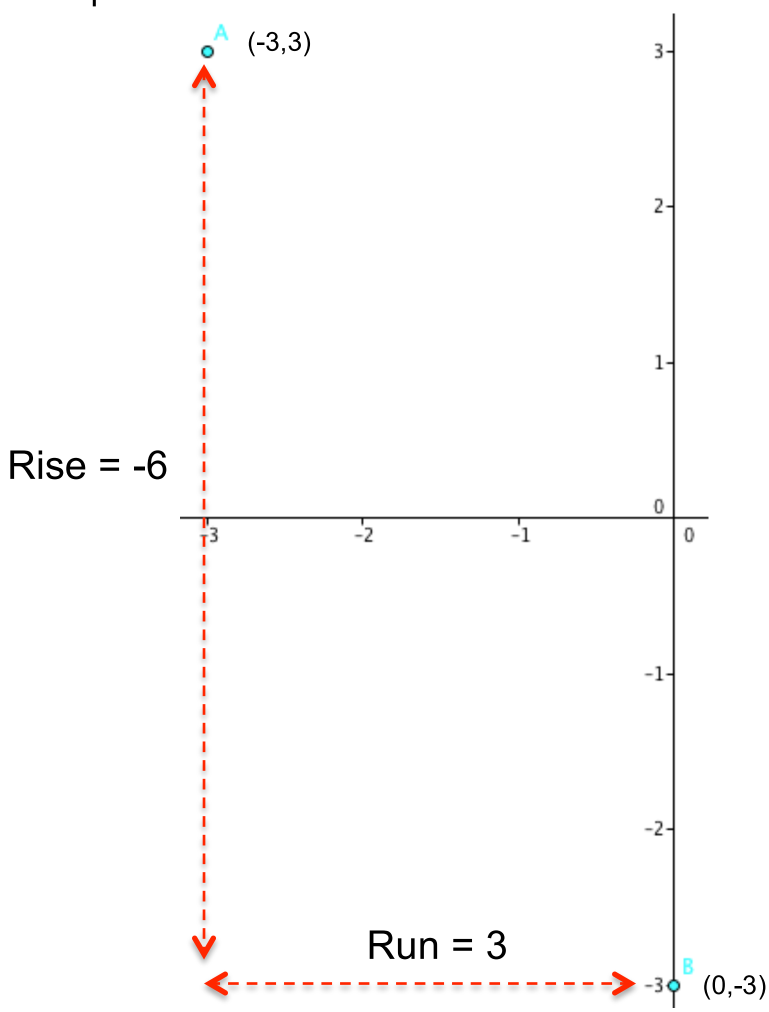
2. For each of the following prompts, create a graph of a function that has the stated properties.

(a) $y = f(x)$ such that

- the average rate of change of *f* on [−3, 0] is −2 and the average rate of change of *f* on [1, 3] is 0.5, and
- the instantaneous rate of change of *f* at x = −1 is −1 and the instantaneous rate of change of *f* at x = 2 is 1.

The intervals given in the first bullet (above) indicate that the graph will be viewed between -3 and 3 as the first interval starts at -3 and the second interval ends at 3. Therefore, the graph will be created based off of the two intervals mentioned.

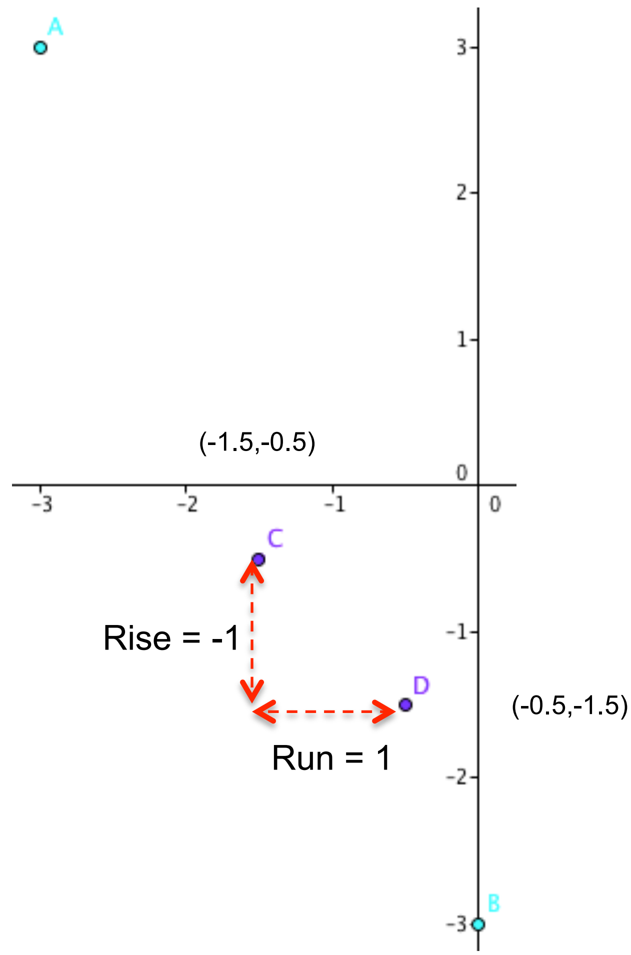
Since we know that the average rate of change (or slope) of *f* on [-3, 0] is -2, we can begin by plotting some points (A and B below) that will create a line with that slope.



Chelsea Smith
MTH 201-05
Portfolio Problem 1

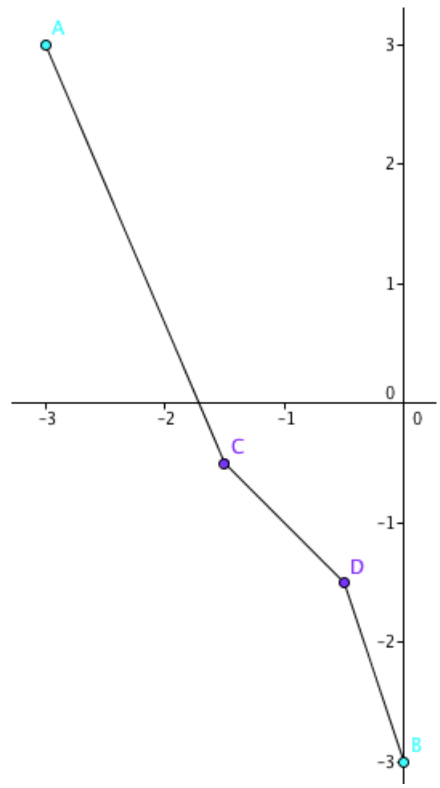
In the above graph, I started by placing point A at (-3,3). Since we know that the distance (run) between x = -3 and x = 0 is 3 (0 – (-3)), we also know that the rise must be -6 in order to give me a slope of -2. So, I placed point B at (0,-3) since y = -3 minus y = 3 gives us a rise of -6.

We are given the instantaneous rate of change of *f* at x = -1 is -1. So, there can be a change in slope at any given interval that includes x = -1, and is between x = -3 and x = 0. Below, I have added on to the graph above by including points C and D that provide and instantaneous velocity of -1 at x = -1.

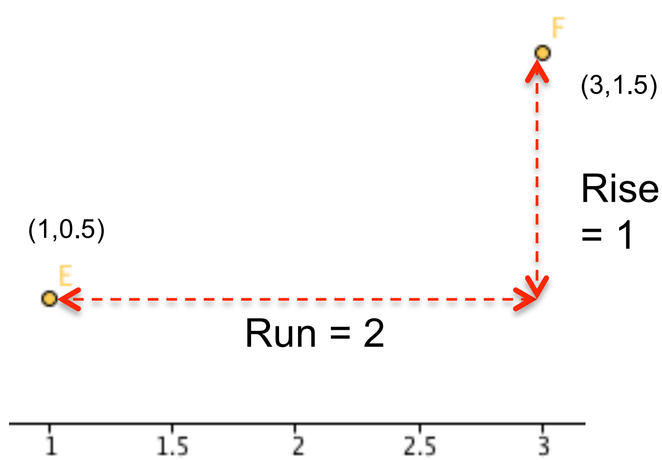


I set points C and D apart by a distance (run) of 1 with C at x = -1.5 and D at x = -0.5. Because the rise has to be -1 in order to provide us with a slope of -1, I placed point C at y = -0.5 and point D at y = -1.5. Now that the criterion on [-3, 0] has been met, we can connect the four points A, C, D, and B. The next page contains a graph to show these new connections.

Chelsea Smith
MTH 201-05
Portfolio Problem 1



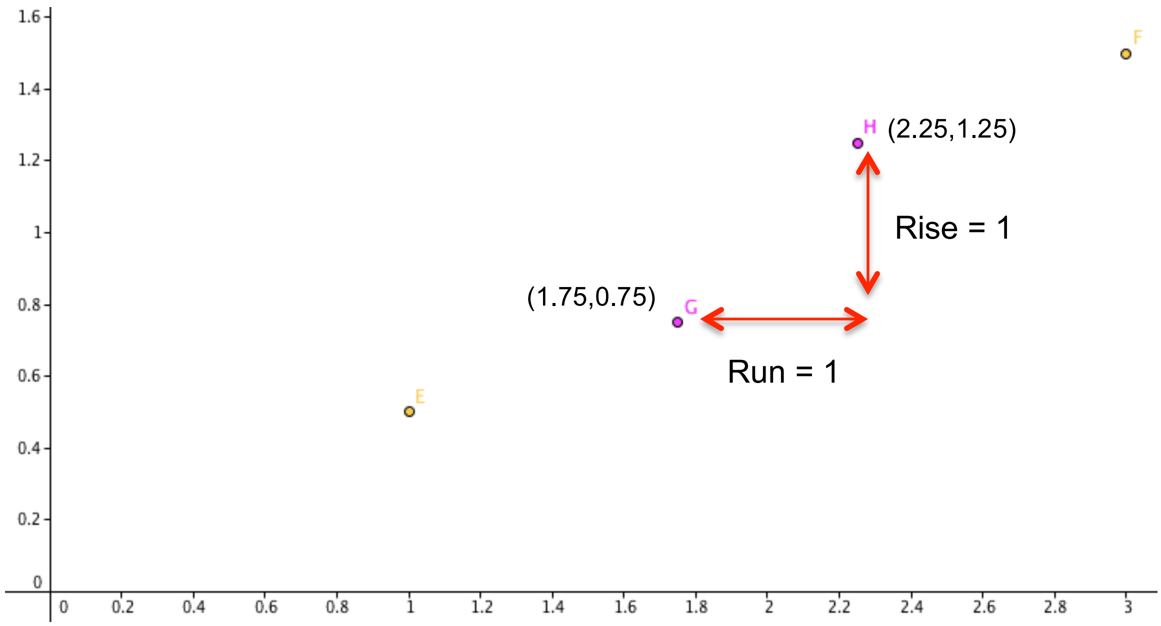
We can now work on creating the right side of the graph where the average rate of change of *f* on [1, 3] is 0.5. Pictured below is the chunk of a graph that fits the given standards.



I built this section of the graph by placing point E at (1,0.5). Because we know that the distance (run) between x = 1 and x = 3 is 2 (3 – 1), we know that the rise must be 1 in order to give me a slope of 0.5. So, I placed point F at (3,1.5) since y = 1.5 minus y = 0.5 gives us a rise of 1.

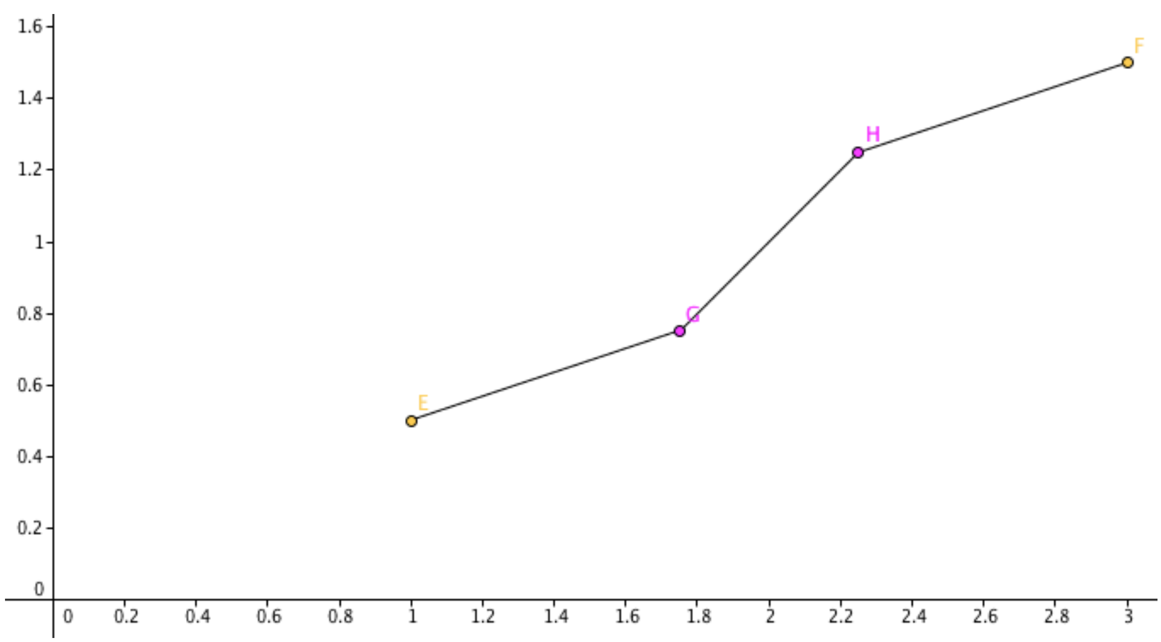
Chelsea Smith
MTH 201-05
Portfolio Problem 1

As given to us in the second bullet, we know that the instantaneous rate of change of *f* at x = 2 is 1. So, we can now do the same to this right portion of the graph as we did with the left side of the graph in order to include this instantaneous velocity piece.



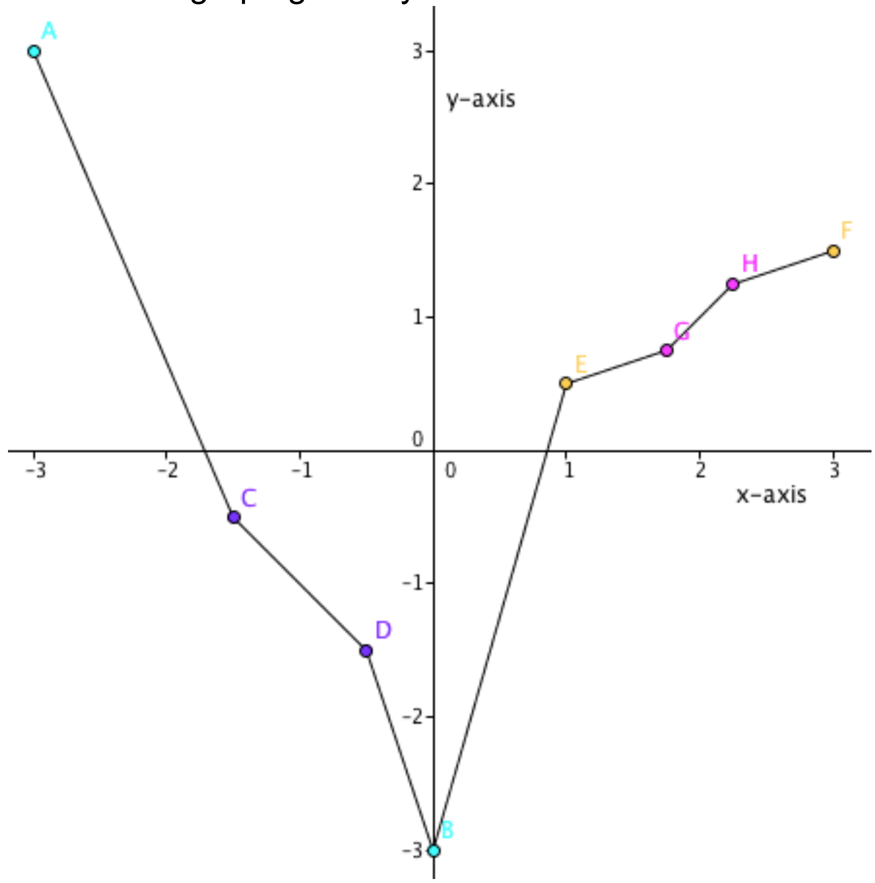
The above graph includes an instantaneous rate of change of 1 at x = 2. I set points G and H apart by a distance (run) of 0.5 with G at x = 1.75 and H at x = 2.25. Because the rise has to be -0.5 in order to provide us with a slope of -1, I placed point G at y = 0.75 and point H at y = 1.25.

Once again, we can connect the four points E, G, H, and F now that we have fulfilled the requirements on the right side of the graph. Here is the graph that shows these connections.

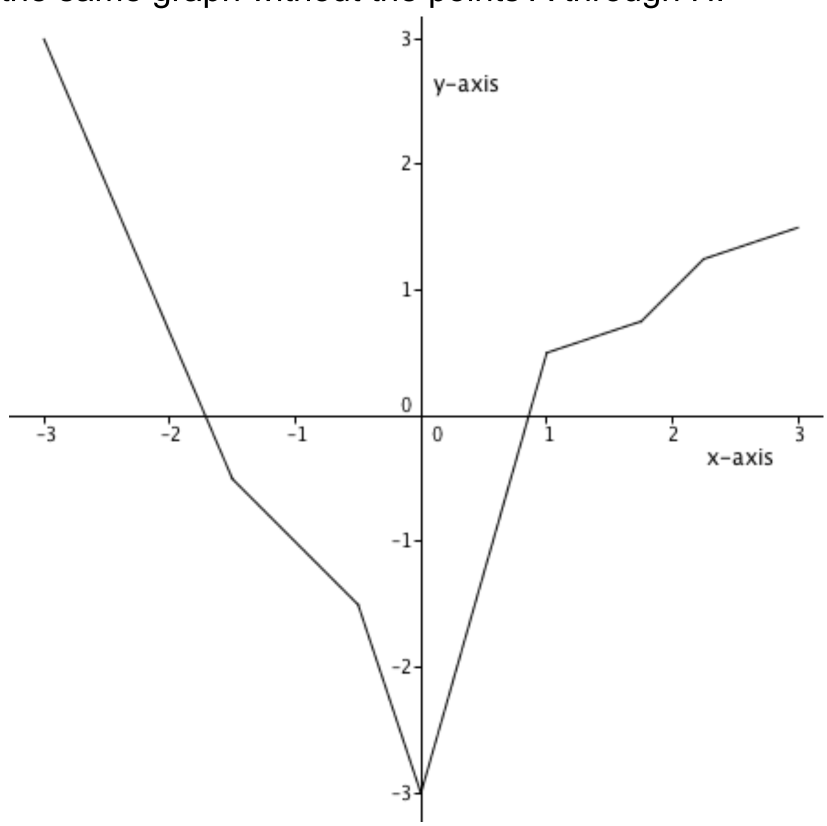


Chelsea Smith
MTH 201-05
Portfolio Problem 1

Below is the resulting graph of the work we did to create the left [-3, 0] and right [0, 3] sides of the graph given by the criteria in the two bullets.



And here is the same graph without the points A through H.

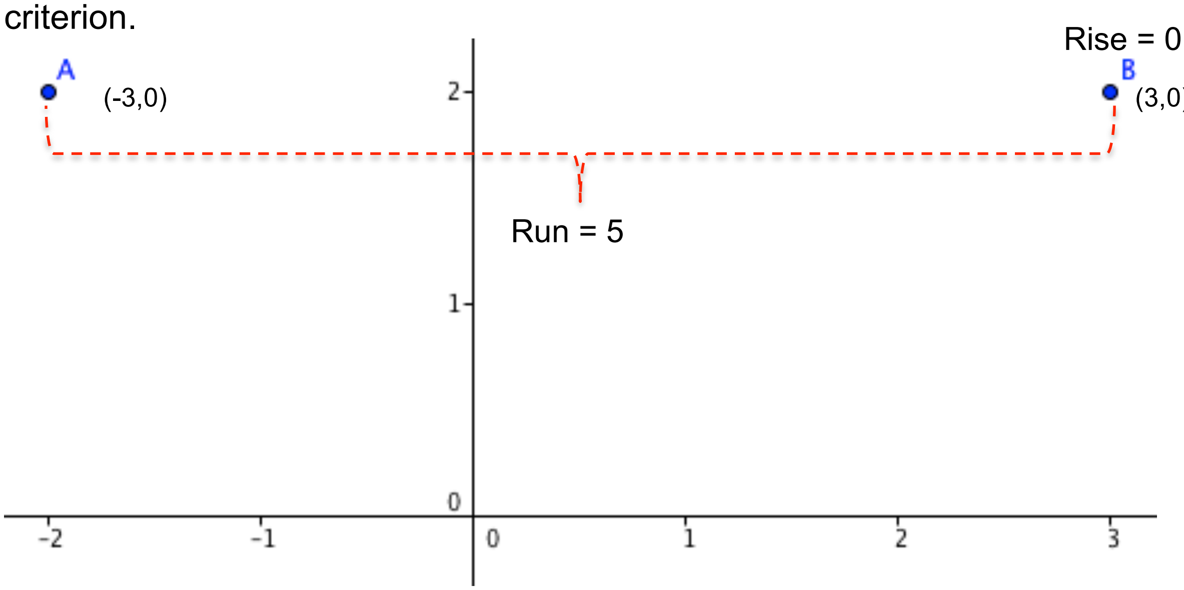


Chelsea Smith
MTH 201-05
Portfolio Problem 1

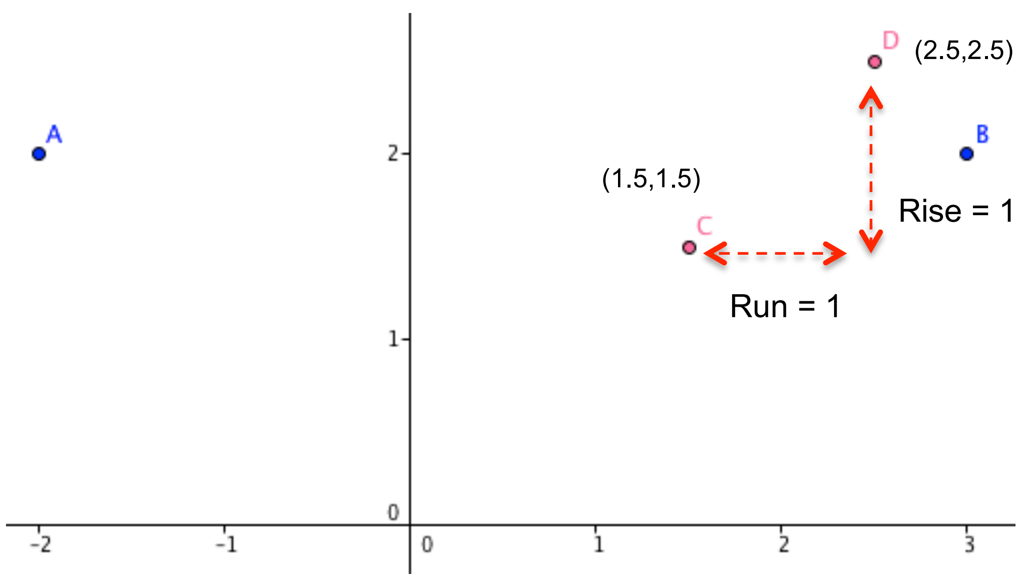
(b) $y = g(x)$ such that

- $\frac{g(3)-g(-2)}{5} = 0$ and $\frac{g(1)-g(-1)}{2} = -1$
- $g'(2) = 1$ and $g'(-1) = 0$

In the first bullet, $\frac{g(3)-g(-2)}{5}$ = 0 indicates that the average rate of change (or slope) of g between x = -2 and x = 3 is 0. The given fraction is simply saying that the y-value at x = 3 minus the y-value at x = -2 divided by the distance of 5 (3 – (-2)) between the two x-values provides us with a slope of 0 (or a horizontal line). Below are possible A and B points to fit this criterion.

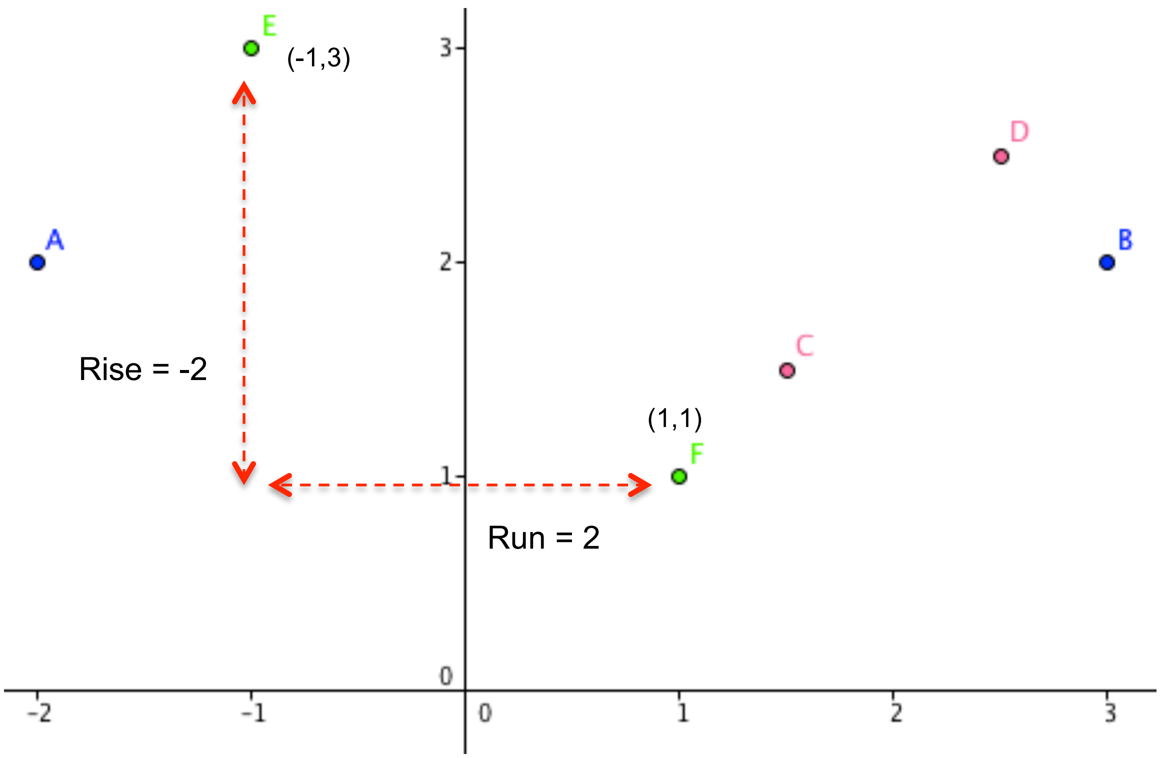


The first part of the second bullet provides us with the instantaneous rate of change of 1 at x = 2 ($g'(2) = 1$). So, we can incorporate this information into the above graph just as we did in part (a). Here is the graph with two additional points, C and D, which add this new information.



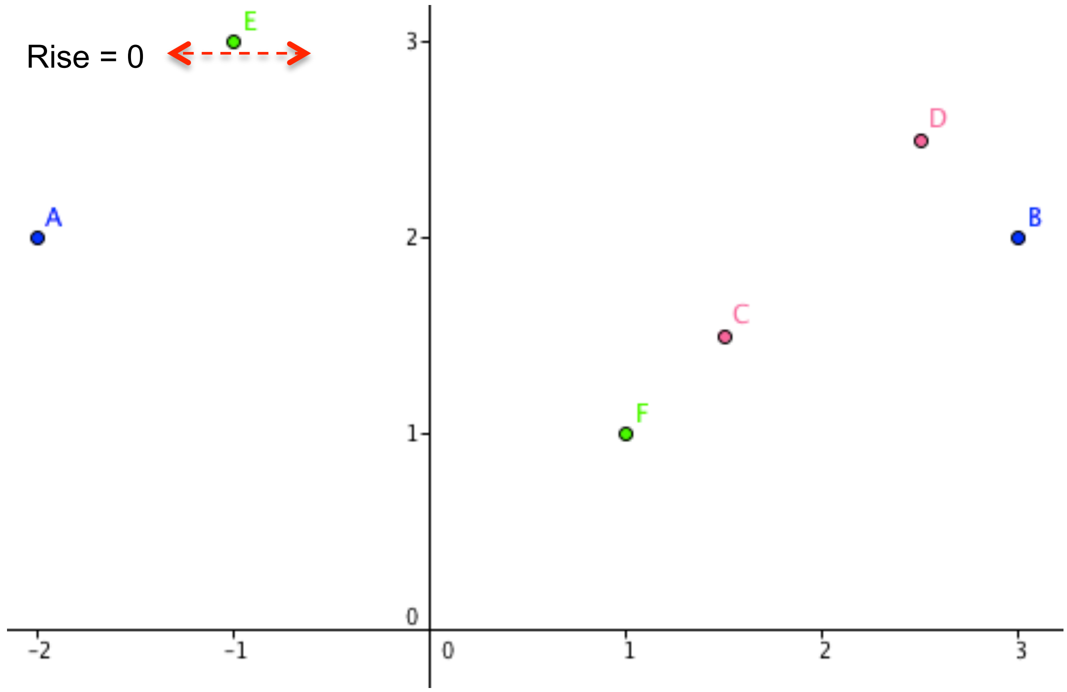
Chelsea Smith
MTH 201-05
Portfolio Problem 1

The second portion of the first bullet gives us the following information: $\frac{g(1)-g(-1)}{2} = -1$. This provides us with the average rate of change of -1 of g between x = -1 and x = 1. Once again, this fraction is telling us that the y-value at x = 1 minus the y-value at x = -1 divided by a distance of 2 (1 – (-1)) between the two x-values gives us a slope -1. Below are some points that incorporate the given information and adds on to the graph above.

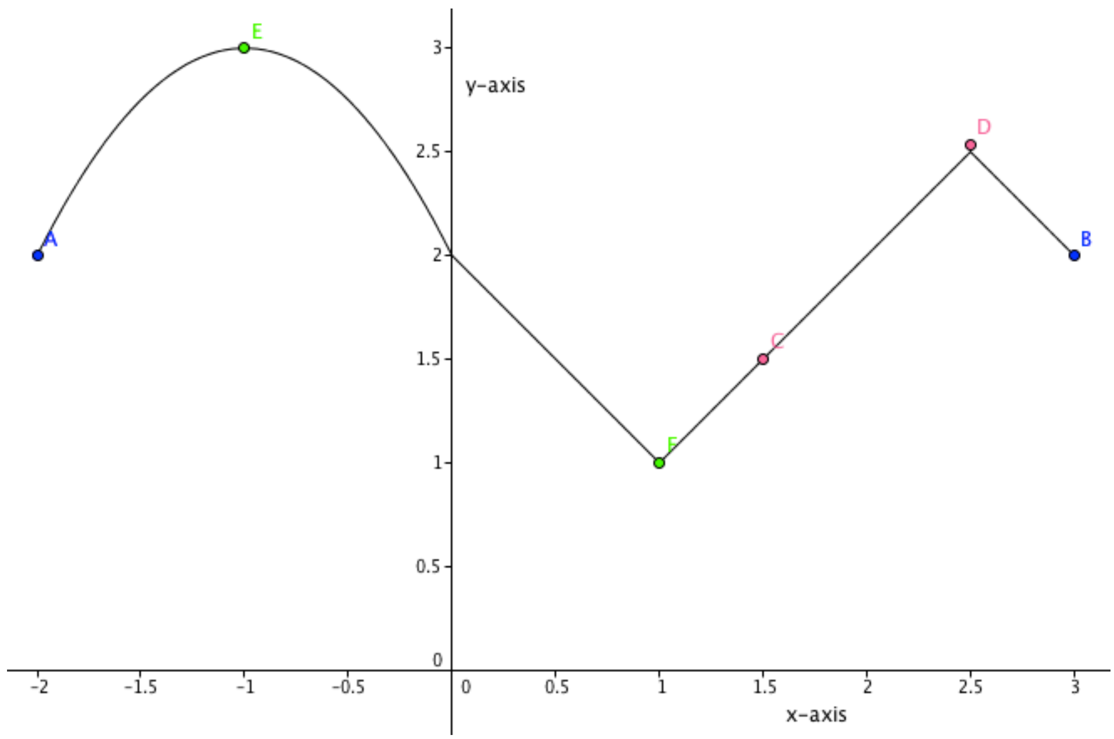


The final chunk of the graph that we need to include is the instantaneous velocity given to us by the last part of the second bullet. $g'(-1) = 0$ says that the slope at x = -1 is 0. A line with a slope of 0 has been added to the graph above at point E. Note that the line below is only representing the slope of 0 that will take place at this point once all of the points are connected. (See the instantaneous velocity of 0 at x = -1 in the second graph on the following page that I created by using a concave down parabola with the peak at x = -1.)

Chelsea Smith
MTH 201-05
Portfolio Problem 1

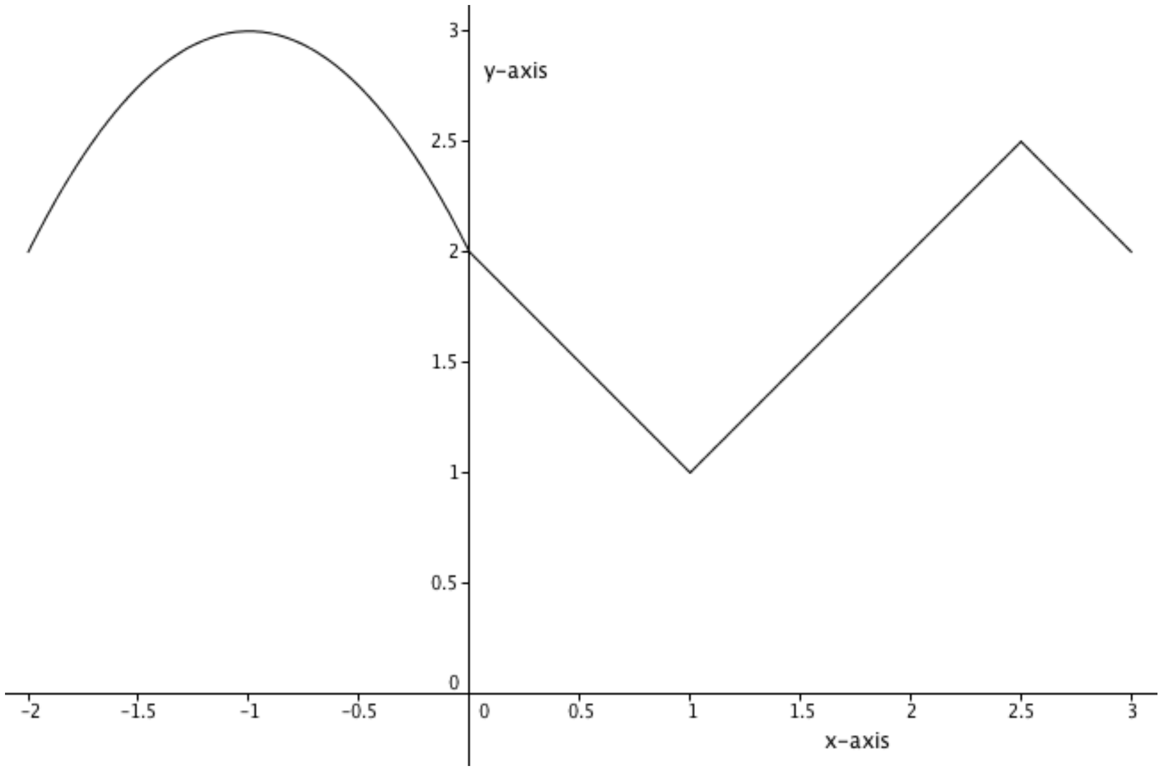


From here we can connect all of the points (A, E, F, C, D, and B) to make a graph that fits the four pieces of given information. Pictured below is the final graph (with points) that includes all of the criteria for part (b).



Chelsea Smith
MTH 201-05
Portfolio Problem 1

Here is the final graph without points A – F.



NOTE: There are many different ways we could have gone about creating the graphs in parts (a) and (b). For example, the graph in part (a) could have been shifted up or down, and the instantaneous rate of change at x = -1 could have lasted over a shorter or longer interval. As long as the average rate of change, instantaneous rate of change, and interval criteria of a function are met, it does not matter how you construct your graph.