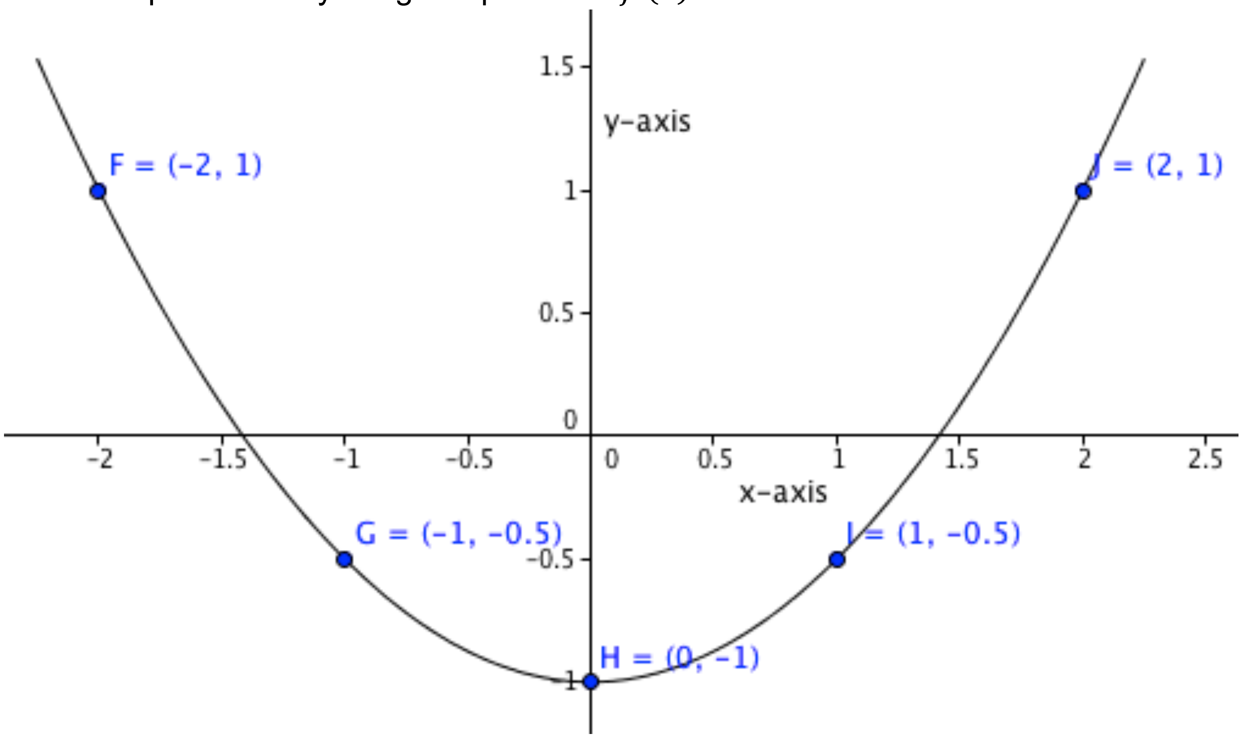
Chelsea Smith
MTH 201 – 05
Portfolio Problem 2

Portfolio Problem

Problem 2: Section 1.4 #1

1. Let f be a function with the following properties: f is differentiable at every value of x (that is, f has a derivative at every point), $f(-2) = 1$, and $f'(-2) = -2$, $f'(-1) = -1$, $f'(0) = 0$, $f'(1) = 1$, and $f'(2) = 2$.

(a) Here is a possible graph of $y = f(x)$. I created this function graph based off of the given point (-2,1) on $f(x)$, and the slopes of -2, -1, 0, 1, and 2 presented by the given points on $f'(x)$.



This graph of $y = f(x)$ fits the given criteria. If you look at each of the six areas of differentiability given above, they help explain why, in fact, the graph meets the criteria.

i) $f(-2) = 1$ → Point F is at (-2,1)
ii) $f'(-2) = -2$ → The slope at x = -2 (F) is -2
iii) $f'(-1) = -1$ → The slope at x = -1 (G) is -1
iv) $f'(0) = 0$ → The slope at x = 0 (H) is 0
v) $f'(1) = 1$ → The slope at x = 1 (I) is 1
vi) $f'(2) = 2$ → The slope at x = 2 (J) is 2

(b) Below is a possible graph of $y = f'(x)$ given the data points above.

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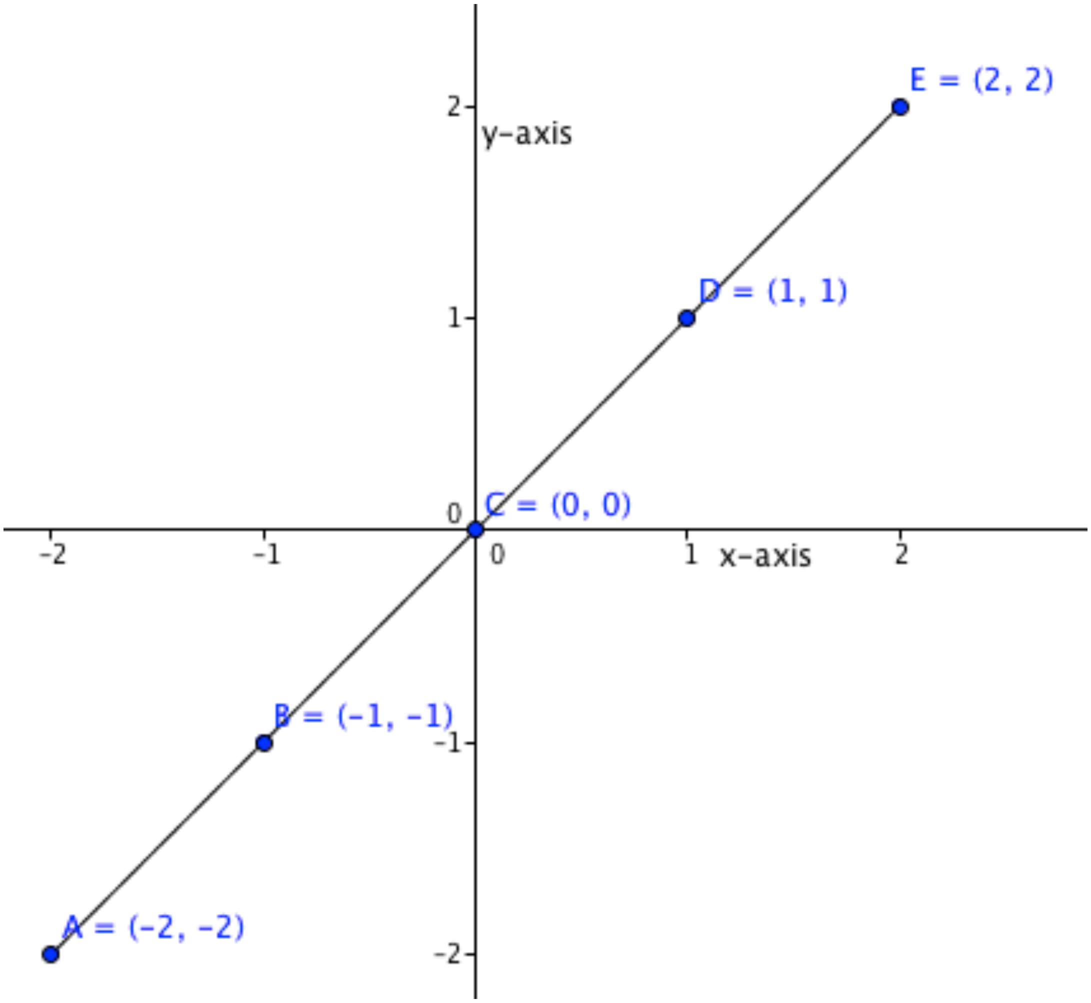

The provided data points given above suggest that the slope of the derivative graph will be 1 (just as the slope of this graph is). This is because both the inputs (x-values) and outputs (y-values) of the derivative function are increasing by 1 from one point to the next. For example, when moving from point A to point B, the y-value increases by 1 from y = -2 to y = -1, and the x-value increases by 1 from x = -2 to x = -1. Using the slope formula, we can show that the slope is, in fact, 1.

$$\frac{-2-(-1)}{-2-(-1)} = \frac{-2+1}{-2+1} = \frac{-1}{-1} = 1$$

(c) A possible formula for $y = f(x)$ is $f(x) = \frac{1}{2}x^2 - 1$.

- In order to create this formula, I took into consideration what *type* of graph $f(x)$ is, its terms of *stretching* or *compressing*, and what its *y-intercept* is. To determine such characteristics, I simply looked at the graph of $f(x)$ I created and used points from the function.
- *Style*: By simply looking at the graph of $f(x)$, you can see that the formula will be quadratic. So, we will begin by using the quadratic

equation $f(x) = ax^2 + bx + c$. Note that *bx* will not be part of this particular function equation.

- *Stretch* or *Compress*: When determining horizontal stretches (vertical compressions) and horizontal compressions (vertical stretches), we need to look at the coefficient *a* in the quadratic equation. Because we don't know what *a* is yet, we must try plugging in a point or two on $f(x)$, as well as the *y-intercept* (which is the variable *c* in the quadratic equation.
- *Y-intercept*: The function created $f(x)$ function has a y-intercept at y = -1.

In the chart below, you can see that *a* is being solved for.
I solved for *a* by plugging in a point on $f(x)$. Note that you would only have to use one point. I just chose to test several points.

x-value ↓ y-value ↓

$$a(x)^2 - 1 = (y)$$

$(-2,1)$	$a(-2)^2 - 1 = (1)$ $4a = 2$ $a = \frac{2}{4} = \frac{1}{2}$
$(-1,-0.5)$	$a(-1)^2 - 1 = (-0.5)$ $1a = \frac{1}{2}$ $a = \frac{1}{2}$
$(1,-0.5)$	$a(1)^2 - 1 = (-0.5)$ $1a = \frac{1}{2}$ $a = \frac{1}{2}$
$(2,1)$	$a(2)^2 - 1 = (1)$ $4a = 2$ $a = \frac{2}{4} = \frac{1}{2}$

Therefore, we can say that the function $f(x)$ has a horizontal stretch (vertical compression) of $\frac{1}{2}$. Because we now have our *a* and c coefficient values determined, we can confirm the formula I gave above:

$$f(x) = \frac{1}{2}x^2 - 1$$

Note that it is important to understand that there are other possible functions and graphs that can fit the criteria given on the first page. This is only one of the many possible functions that fits the given points and slopes.

We must lastly use the limit definition of the derivative in order to determine the corresponding formula for $y = f'(x)$.

$$f(x) = \frac{1}{2}x^2 - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{2}(x+h)^2 - 1 - \left(\frac{1}{2}x^2 - 1\right)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{2}(x+h)^2 - 1 - \left(\frac{1}{2}x^2 - 1\right)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{2}(x^2 + 2xh + h^2) - 1 - \frac{1}{2}x^2 + 1}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{\frac{1}{2}x^2} + xh + \frac{1}{2}h^2 \cancel{- 1 - \frac{1}{2}x^2 + 1}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{xh + \frac{1}{2}h^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}\left(x + \frac{1}{2}h\right)}{\cancel{h}}$$

$$f'(x) = \lim_{h \to 0} x + \frac{1}{2}(0)$$

$$f'(x) = x$$

As h approaches 0, all that remains is x.
Therefore, the f'(x) = x and has a slope of 1.
If you look at the derivative graph above, we can confirm that this function is correct.